

About Image FFT Examples

Upper Left - Aliasing.

A triangle and the power spectrum of its Fourier transform are displayed in the boxes. The power spectrum is then periodically replicated to illustrate the 'wrap around artifact' characteristic of aliasing. Because of the triangle's sharp edges, its power spectrum is severely aliased.

Upper Right - Motion Blur Removal Using Deconvolution.

(Excerpt directly from *Optimized Fast Hartley Transform for the MC68000 with Applications in Image Processing*, Arlo Reeves' MS Thesis at Dartmouth's Thayer School of Engineering, April 1990):

Figure a [upper left] shows an image exhibiting a horizontal blurring similar to that obtained when photographing a quickly moving object with a slow shutter speed. As the planetary probe Voyager ventured to the outer planets, the waning sunlight demanded longer exposure times and such motion blur affected every image recorded. Fortunately, if the nature of the motion is known and the object being imaged is at a constant distance from the camera (a good approximation for planetary imaging), motion blurring can be easily corrected.

Motion blur can be modeled as the convolution of the image with a line whose length and orientation reflect the camera's shutter speed and the object's direction of motion, respectively (Figure a [upper left] was generated by convolving the original image with a horizontal line 13 pixels long). Since convolving in the space domain corresponds to multiplying in the frequency domain, the Fourier transform of the undegraded image is multiplied by the Fourier transform of the line to produce the power spectrum of Figure b [upper right]. The spacing and orientation of the zeroes of Figure b belie the length and orientation of the blurring line, allowing one to generate its Fourier transform independently. Dividing the Fourier transform corresponding to Figure b by the Fourier transform of the blurring line gives the power spectrum of Figure c [lower right]. Although some of the original power spectrum is lost, inverse transformation shows this deconvolution operation successfully restores the original image in Figure d [lower left].

Note that the vertically oriented zeros of the blurring line's power spectrum would cause the quotient transform of Figure c to 'blow up' along these lines. Normally, the inverse filter is apodized to avoid this effect. With the limited dynamic range of our integer calculations, these singular quotients occur more frequently than when real numbers are used and consequently more of the quotient transform is lost (the white area in Figure c). While this limits the use of deconvolution in *Image*, it is still of practical use as this example shows.

Lower Left - Myofibril Structure.

(Excerpt again):

Skeletal muscle is made up of long, thin cells each containing many myofibrils. Myofibrils in turn contain long myosin and actin molecules forming a regular bundles of

thick and thin filaments, respectively. When a skeletal muscle cell is viewed in cross section, as in the TEM image of Figure a [upper left], the regular arrangement of the thick filaments becomes apparent only after some scrutiny. The regularly spaced spectral features of Figure a's power spectrum, shown in Figure b [upper right], reveal that there is in fact a great deal of regular structure in the original image.

Using the Threshold Zero and Filter operations, the dominant peaks of the power spectrum in Figure b are isolated, producing the power spectrum in Figure c [lower right]. The inner ring of six spectral peaks corresponds to the regular thick filament spacing, while the outer ring corresponds to the higher frequency thin filament spacing. The inverse transform of this modified power spectrum is shown in Figure d [lower left], revealing the basic periodic structure of Figure a, including that of the thin filaments which could not be seen in the original image.

Lower Right - T4 Bacteriophage Tail Structure Enhancement.

(Excerpt again):

The T4 bacteriophage 'represents the extreme in structural complexity among bacterial viruses' and has been widely studied by biologists [29]. Its tail structure, a detail of which is shown Figure b [upper left] has been modeled as a tube of helically interwoven proteins. The structure apparent in most TEM images, however, makes the T4 tail appear like a stack of disks viewed on edge. Consequently, the power spectrum of Figure b, shown in Figure c [upper right] is dominated by the frequency of the disk spacing. Since both the image and its power spectrum are spatially calibrated, *Image* allows one to easily establish that the disk spacing is 3.9 nm by simply moving the mouse cursor over the power spectral peaks.

Also discernible in the power spectrum of Figure c are spectral features of frequency similar to the disk spacing, but oriented at an angle. These have been enhanced in Figure d [lower right] by multiplying all other spatial frequencies by a factor of 0.25 using *Image*'s Pass operation. Inverse transformation yields Figure e [lower left], which more clearly reveals the helical structure of the T4 tail.

While the emphasis of specific spatial frequencies has been used to advantage here, it should be emphasized that this technique can be exploited to provide 'enhancements' that are unrealistic. Microscopists generally emphasize or filter out complete annuli in the frequency domain, avoiding the preferential treatment otherwise given to spatial frequencies of a particular orientation. This technique may also be easily applied using *Image*'s FFT extensions.